Do Latin American Central Bankers Behave Non-Linearly? The Experiences of Brazil, Chile, Colombia and Mexico*

Luiz de Mello†, Diego Moccero‡ and Matteo Mogliani§

†OECD
‡European Central Bank
§Banque de France

October 2012

Abstract

This paper contributes to the empirical literature on inflation zone targeting by estimating monetary reaction functions in a non-linear cointegration framework for Brazil, Chile, Colombia and Mexico. Evidence shows that central banks respond linearly to deviations of expected inflation from the target (the inflation gap) in Brazil and Chile. As the inflation gap widens, policy responses become weaker in Colombia and Mexico, a finding that most probably reflects a history of adverse supply shocks rather than a lack of resolve from the monetary authorities. Non-linearity is also found in the central bank’s response to fluctuations in the exchange rate in Brazil and Colombia.

Keywords: inflation zone targeting, reaction function, non-linear cointegration, smooth transition models.

JEL classification number: C22, E52, O54.

*We are indebted to Peter Jarrett, Elena Rusticelli, José Sanchez Fung, Lukas Vogel and the participants in the 24th Latin American Meeting of the Econometric Society (Rio de Janeiro, 20-22 November, 2008), the 18th International Tor Vergata Conference on Money, Banking and Finance (Rome, 2-4 December, 2009), the 27th Symposium on Money, Banking and Finance (Bordeaux, 17-18 June, 2010) and in seminars and workshops at the Paris School of Economics for helpful comments and discussions. Special thanks to the editor, two anonymous referees, Peter C. B. Phillips, In Choi and Jörg Breitung. The views expressed in this paper are those of the authors and do not necessarily reflect those of the BdF, the ECB or the OECD.

†OECD Office of the Secretary-General, 2 rue Andre Pascal, 75755 Paris Cedex 16 (France). Email: luiz.demello@oecd.org

‡Corresponding author, European Central Bank, Kaiserstrasse 29, 60311 Frankfurt am Main (Germany). Email: diego.moccero@ecb.int

§Banque de France, 46-1383 DGEI-DCPM-DIACONJ, 31 Rue Croix des Petits Champs, 75049 Paris Cedex 01 (France). Email: matteo.mogliani@banque-france.fr.
1 Introduction

A large number of developed and emerging-market economies have adopted inflation targeting as the institutional framework for monetary policy making. In particular, many central banks, especially in small open economies target a range for inflation, rather than a central objective, which allows some necessary flexibility in response to economic shocks.¹ Developed countries that have adopted inflation zone targeting include Australia, Canada, New Zealand, Norway, the United Kingdom and Sweden. In Latin America, central banks in Brazil, Chile, Colombia and Mexico also target inflation within a (symmetrical) tolerance band around the central target.

Orphanides and Wieland (2000) argue that focusing on containing inflation within a target range entails different optimal responses to shocks than in the case of point inflation targeting. These differences in policy responses suggest that monetary reaction functions under inflation zone targeting may be non-linear: reactions to deviations of inflation from the central target may become stronger as inflation approaches the boundaries of the target zone. This non-linearity in the monetary reaction function arises as a result of deviations from the conventional minimization of quadratic loss functions subject to linear Phillips curves and aggregate demand schedules, as in Svensson (1997) and Ball (1999). In particular, the loss function of the central bank may assign different values to inflation deviations that lie outside the target zone and within the admissible range. Alternatively, it is possible that the Phillips curve may reflect more complex price-setting mechanisms than those subsumed in a linear specification. For example, inflation may respond to the output gap only when it falls outside a given range of values.² Svensson (1997) and Ball (1999) also show that uncertainty in supply and demand has a role to play in a non-linear framework. They show that it is optimal for the policymaker to start responding

¹For example, Erceg (2002) shows that the optimal width of an inflation band in an open economy increases with the strength of terms of trade shocks. Similarly, Mishkin and Westelius (2008) find that more uncertain inflation leads to wider target ranges. See Castelnuovo et al. (2003) for a discussion of the pros and cons of ranges versus point inflation targets and for a review of operational practices for inflation targeting in developed countries.

²Evidence seems to support this hypothesis for the United States. Barnes and Olivei (2003) find that inflation does not react to the unemployment gap over a range of values, but a significant trade-off emerges outside this range.
to inflation deviations from the target within the admissible zone and to do so more aggressively if inflation drifts outside the zone.\(^3\)

Against this background, this paper tests empirically for the presence of non-linearity in the monetary reaction functions of four Latin American inflation targeting countries: Brazil, Chile, Colombia and Mexico. If the model proposed by Orphanides and Wieland (2000) for inflation zone targeters is correct, monetary policy should become more responsive to deviations of inflation from the central target as these deviations widen and the probability of breaching the target zone increases. There is a fairly large body of research on policy rules in Latin American countries, but emphasis has so far been placed on the estimation of linear, rather than non-linear, reaction functions (Corbo, 2002; Schmidt-Hebbel and Werner, 2002; Minella et al., 2003; Cerisola and Gelos, 2005; de Mello and Moccer, 2009, 2011). Instead, the literature on developed countries has provided empirical evidence in favour of the non-linearity hypothesis, as suggested by the inflation zone targeting theory, at least for the United States (Tachibana, 2008), Japan (Tachibana, 2006) and the United Kingdom (Boinet and Martin, 2008).\(^4\)

Non-linearity is tested in this paper in an unrestricted monetary reaction function setting. Accordingly, the functional specification of the reaction function does not depend on the specific parameterization of the theoretical model describing central bank behavior. The econometrician is agnostic about the source and type of non-linearity that may emerge in the estimation of the policy rule. Non-linearity may arise in the monetary authority’s reaction to changes in inflation or deviations of inflation expectations from the target. Regime switches, which give rise to non-linear behavior, may be discontinuous, as in the case of threshold models (Tsay, 1998; Hansen and Seo, 2002), or continuous, as in smooth transition models (Granger and Teräsvirta, 1993; Teräsvirta, 2006). This agnosticism is convenient, because it has often been argued that parametric restrictions imposed on the functional specification of the reaction functions on the basis of structural models of central bank behavior may generate misspecification biases (Kim et al., 2005).

---

\(^3\)This is because, under uncertainty, there is always a probability that a shock will drive inflation outside the target zone, prompting an immediate policy reaction.

\(^4\)Tachibana (2006, 2008) notes that both the United States and Japan have implemented \textit{implicit} inflation zone targeting since neither the US Fed nor the Bank of Japan has adopted explicit inflation targeting regimes.
We focus on the post-1999 period, when fully-fledged inflation zone targeting was adopted as the institutional framework for monetary policy in the four countries under examination. We test for linear cointegration among the variables of interest, as well as for neglected non-linearity in the linear specification of the policy rules. We report the results of the estimation of linear and, where applicable, non-linear reaction functions, while controlling for possible endogeneity among the regressors. We estimate smooth-transition models, allowing for non-linearities to arise both in the inflation gap (i.e., deviations of expected inflation from the central target) and the exchange rate.

We also contribute to the empirical literature by accounting for non-linearity in central bank behavior using integrated variables. Most of the econometric techniques available to date for testing for non-linearity require the time series to be stationary. This is a restrictive requirement for emerging-market economies, where the relevant variables often exhibit unit roots, which calls for the use of non-linear cointegration techniques that allow for integrated series.

Our main findings are as follows. First, we find evidence of neglected non-linearity in the monetary reaction functions in Brazil, Colombia and Mexico, but not in Chile. Second, estimation of smooth-transition models for Brazil, Colombia and Mexico shows that the central bank’s response to the inflation gap is linear in Brazil, because the estimated policy response is statistically insignificant in the non-linear part of the model. As the inflation gap widens, policy responses become weaker in Colombia and Mexico, a finding that can be attributed to the magnitude of the adverse supply-side shocks that have hit these economies. Tackling a sharp rise in inflation due to a large adverse shock might require a significant loss in output. Instead, the monetary authorities might focus on the second-round effects of such shocks, rather than dealing with their first-round effects on inflation. Third, monetary reactions to fluctuations in the exchange rate are non-linear in Brazil and Colombia. A nominal exchange-rate depreciation triggers a tightening of monetary policy, but this response weakens as inflation deviates from the target. We attribute this finding to the fact that, under inflation zone targeting, the monetary authorities may respond to exchange-rate fluctuations that could create inflationary pressures only insofar as inflation remains within the target range. Finally, the threshold for regime changes, i.e., the value for the inflation gap that prompts the authorities to react more strongly to deviations
of inflation from the target, is positive for Mexico and for Colombia. For Brazil, the threshold is negative, suggesting that the central bank may have tried to anchor inflation expectations at a level that has been on average somewhat lower than the central target, possibly in an attempt to build credibility in the policy regime.

The paper is organized as follows. Section 2 briefly reviews the empirical literature on inflation targeting in Latin America. Section 3 describes the data and reports the results of the unit root tests. Section 4 reports the results of the linear and non-linear cointegration tests, and discusses the cointegration equations. Section 5 concludes.

2 The literature on Latin America: A brief summary

Linear reaction functions have been estimated for the four countries under examination. Minella et al. (2003) estimate a forward-looking reaction function for Brazil in levels and augment it by changes in the nominal exchange rate. On the basis of OLS regressions, they report a greater-than-one coefficient on the inflation gap, a relatively weak response to the output gap and a strong reaction to changes in the exchange rate. Corbo (2002) reports the results of the estimation of linear reaction functions for a number of Latin American countries, including Chile and Colombia. In both countries, the estimated central bank response to changes in the inflation gap is statistically significant, correctly signed and greater than one in the inflation-targeting period, on the basis of GMM estimations. Schmidt-Hebbel and Werner (2002) estimate linear exchange-rate-augmented reaction functions for Brazil, Chile and Mexico. The authors report mixed findings on the sign, significance and magnitude of the coefficients on the inflation gap. Monetary policy reaction to the exchange rate is found to be weak.

Most of the empirical literature for Latin America report near-unity coefficients on the lagged interest rate, a finding that may well be due to the presence of unit roots in the interest rate series. The policy rules are estimated for variables in levels, and little attention

---

5 Monetary policy reaction functions that include the exchange rate have been estimated by Mishkin and Savastano (2001) and Mohanty and Klau (2005), among others. The monetary authorities are hypothesised to care about the exchange rate and its variability because of their impact on inflation (via relative prices and expectations, for instance); on the performance of the external sector, investment and growth (via trade competitiveness); on financial and public debt sustainability (via balance-sheet effects) and on the development of foreign-exchange and capital markets. See Ho and McCauley (2003) for a comprehensive study including both developed and developing country inflation targeters.
is paid to the unit root properties of the data. This is essentially because motivation for
the empirical analysis comes from the estimation of Taylor rule-type restricted reaction
functions, whose specification is borrowed directly from the literature. An assessment of
the time-series properties of data by de Mello and Moccero (2009, 2011) nevertheless shows
that inflation, expected inflation and the interest rate all have unit roots in the four coun-
tries under examination in the post-1999 inflation-targeting period. The authors therefore
estimate both unrestricted reaction functions in an error-correction setting (de Mello and
Moccero, 2009) and restricted functions in first differences derived from reduced-form New
Keynesian models (de Mello and Moccero, 2011). Linear cointegration tests suggest that
the interest rate, inflation and expected inflation move around a common stochastic trend
in these countries. They interpret this finding as evidence of the existence of a stable
long-run monetary reaction function.

There have been limited attempts to test for non-linearity in the policy rules for the
countries under examination. Corbo (2002) augment the reaction functions to include the
square of the inflation gap or interaction terms between the arguments of the reaction
function and a dummy variable identifying periods when the output gap was negative. In
neither case is there evidence of non-linearity in the reaction function, at least in the case
of Chile.

In what follows, we depart from the literature in two main ways. First, we discuss
the unit root properties of the data and test for linear cointegration among the variables
included in the monetary reaction functions. We then test for the presence of neglected
non-linearity in these linear models. Second, we estimate non-linear reaction functions
by using a specific class of smooth-transition models that does not require stationary
series. In doing so, we allow for non-linearity to emerge from the existence of different
policy regimes, while explicitly modeling transition across policy regimes, rather than from
the ad hoc augmentation of the reaction function with non-linear terms. By focusing on
unrestricted policy rules, we are agnostic about parameter restrictions imposed on the
estimating equation, as mentioned above.
3 Data and unit root tests

3.1 Data

Our empirical analysis focuses on four Latin American inflation targeters: Brazil, Chile, Colombia and Mexico. Our data set includes monthly observations available from these countries’ central banks for the interest rate, inflation expectations, the inflation target and the exchange rate. The sample period differs across countries, depending on the availability of information on survey-based inflation expectations, starting in July 2001 for Brazil, September 2001 for Chile, September 2003 for Colombia and November 2000 for Mexico. The end of the sample is December 2010 for all countries.

The sample period coincides by and large with the adoption of fully-fledged inflation targeting in Brazil (July 1999), Chile and Colombia (September 1999) and Mexico (January 1999). Chile formally adopted inflation targeting in 1990 together with *de jure* central bank independence, but it was not until September 1999 that exchange rate targeting was formally abandoned. The country announced an inflation target range of 2-4% starting in 2001. Brazil and Colombia adopted inflation targeting following the floating of the *real* in January 1999 in Brazil and the abandonment of exchange rate targeting in Colombia in September 1999. In Brazil, the monetary authorities changed the width of the tolerance band around the central inflation target on few occasions and the current target range is 2.5-6.5% (since 2006). In Colombia, the authorities moved from a central target to a range target in 2003. The range was changed on several occasions, standing since 2010 at 2-4%. Mexico scrapped a narrow exchange rate band in 1995, but a gradual transition to explicit inflation targeting began in earnest in 1998. The country switched from a point to a 2-4% range inflation target in 2003.

The interest rate is defined in nominal terms as the annualized SELIC rate for Brazil, the TPM rate for Chile, the 90-day deposit (CDT) rate for Colombia and the yield on 28-day CETES bonds for Mexico. These are the policy rates for Brazil and Chile, where monetary policy is conducted primarily through open-market operations. For Colombia and Mexico, we follow the literature and use a money-market rate for Colombia and the rate of return on the most liquid, shortest-term government bond in Mexico, where
monetary policy was conducted through quantitative bank reserve targets (corto) during most of the period of analysis.\textsuperscript{6}

Expected inflation is defined as the 12-month-ahead expected consumer price inflation (measured by the IPCA index for Brazil, the IPC indices for Chile and Colombia, and the INPC index for Mexico) available from market surveys conducted by each country’s central bank.\textsuperscript{7} Market surveys are conducted among key financial institutions and consulting firms in the four countries. The surveys include about 90 respondents in Brazil, 35 in Chile, and 30-40 in both Colombia and Mexico, depending on the survey wave.\textsuperscript{8} On the basis of these data, the inflation gap was computed as in de Mello and Moccero (2009). An implicit monthly target was calculated by linearly interpolating the end-year targets. In doing so, for example, the implicit monthly targets for January through November for any given year were defined as the fitted values of a line joining the end-year targets. Deviations of expected inflation from the target were then computed using 12-month leads of the implicit monthly targets.

The exchange rate is defined in units of domestic currency per U.S. dollar. The output gap was computed as the log difference between the actual and the HP-filtered (seasonally-adjusted) industrial production index (the IMACEC, which is an economy-wide activity, rather than industrial production, indicator is used in the case of Chile). The interest rate, expected inflation and exchange rate for the four countries under consideration are depicted in Figure 1.

\begin{center}
[Figure 1 about here]
\end{center}

\textsuperscript{6}For a detailed description of monetary-policy instruments in these countries, see Figueiredo et al. (2002) for Brazil; Cifuentes and Desormeaux (2005) and Loayza and Schmidt-Hebbel (2002) for Chile; Uribe (1999), Vargas (2005), Melo and Riascos (2004) and Clavijo (2004) for Colombia; and Ramos-Francia and Torres (2005) and Central Bank of Mexico (2007) for Mexico.

\textsuperscript{7}While the target horizon of the monetary authorities in Chile and Colombia is about two years, there is no explicit reference to a specific target horizon in the case of Brazil and Mexico. However, these central banks have tended to highlight in their communication with the public the behavior of 12-month ahead, survey-based measures of inflation expectations (see, for example, Central Bank of Mexico, 2003).

\textsuperscript{8}It is important to note that central banks might not react only to market expectations, but also to their own inflation projections. Indeed, forecasts of consumer inflation rates are published by central banks in their Inflation Reports in the four countries under consideration. However, two features make these forecasts unsuitable for use in econometric analysis: \textit{i}) they are only available to the public on a quarterly basis, while the monetary authorities meet eight times per year in Brazil and Mexico, and every month in Chile and Colombia. This mismatch in the data frequency may result in poor estimates of the policy reaction functions; and \textit{ii}) inflation forecasts are reported for a fixed target date horizon only (end-year inflation rates for the next two years). This implies that the length of the forecast window becomes shorter within a cycle of forecasts and then reverts back to the initial length to begin another forecast cycle. This feature reduces the comparability of adjacent surveys.
3.2 Unit root tests

Most of the empirical literature on monetary reaction functions for emerging-market economies is agnostic about the time-series properties of the data. Reaction functions are estimated without testing for the presence of unit roots in the relevant series. We depart from this tradition and start by testing for the presence of unit roots using the Elliott et al. (1996) $ADF_{GLS}$ test, which is more efficient than other tests under the hypothesis of normal residuals. The optimal number of lags is selected on the basis of the Schwartz information criterion, starting with a maximum of 12 lags and testing for normality of the residuals. When the residuals are found to be non-normal for all lags, the Phillips and Perron (1988) $Z_t$ test is used. In those cases, the optimal number of lags is chosen using the Newey-West truncation lag selection criteria.

On the basis of the results reported in Table 1, it appears that the null hypothesis of unit roots cannot be rejected for all variables, except the output gap, which was found to be stationary in levels in all countries. All in all, the results of the unit root tests suggest that the variables included in the policy rules should be tested for cointegration, while considering at the same time the stationary nature of the output gap.\(^9\) These findings also cast doubt over the appropriateness of including the output gap in monetary reaction functions estimated in levels, without relying on an appropriate econometric technique to deal with the different order of integration of the variables. In this paper we test for and estimate cointegrating relationships, while taking into account the stationary nature of the output gap. Another way to deal with the different order of integration of policy interest rates and the output gap in developing countries has been the estimation of monetary reaction functions in first differences. In this respect, evidence seems to suggest

\(^9\)There is some debate about whether or not inflation (and inflation expectations) can be integrated of order one. Some interpret the unit root finding as central bank failure to anchor inflation (expectations) around point inflation targets, even in the long run. In our view, the unit root finding owes much to the high share of food and energy in the consumption basket of developing countries, and substantial supply-side shocks which are characteristic of small open economies. Moreover, monetary authorities in these countries have been highly successful in keeping inflation rates within the inflation bands, over most of the inflation targeting period (see de Mello and Moccero, 2009).
that monetary policy has been countercyclical in these countries since the adoption of the inflation targeting (see de Mello and Moccero, 2009, 2011).\footnote{This is a main difference with respect to monetary reaction functions estimated for developed economies, where both the policy interest rate and the output gap are assumed to be stationary.}

### 4 Monetary reaction functions: The cointegration equations

Because most of the variables of interest are found to have unit roots, the econometric theory of non-stationary variables suggests to test for linear cointegration among the interest rate, the inflation gap and the nominal exchange rate in the four countries under examination and to estimate a cointegrating equation defined as follows:

\[
r_t = \alpha + \delta t + \beta_1 (E_t \pi_{t+12} - \bar{\pi}_{t+12}) + \beta_2 e_t + u_t,
\]

(4.1)

where $\alpha$ is the constant, $t$ is a linear trend, $r_t$ is the interest rate, $E_t$ is the expectation operator, conditional on the information set available at time $t$, $\pi_t$ denotes inflation, $\bar{\pi}_{t+12}$ is the 12-month-ahead inflation target, $e_t$ is the nominal exchange rate, and $u_t$ is an error term. A linear trend is included in the cointegrating equation, because the policy interest rate and the exchange rate exhibit a downward trend during the estimation period (see Figure 1).

However, in order to estimate an economically interpretable Taylor rule, we need to deal with the stationary nature of the output gap both in testing and estimating the monetary reaction functions. To do so, and in order to report and analyze cointegration coefficients which are comparable across linear and non-linear estimates, we work with the adjusted version of cointegrating regressions including I(0) variables suggested by Park and Phillips (1989) and Chang et al. (2001). For both theoretical and practical reasons, which will be clarified in the next sections, we implement the fully-modified OLS estimator (Phillips and Hansen, 1990) in both the linear and non-linear frameworks. Hence, Equation (4.1) is rewritten in such a way that it includes the stationary output gap without violating the
asymptotic properties of the estimator, such that:

\[ \tilde{r}_t = \alpha + \delta t + \beta_1 (E_t \pi_{t+12} - \bar{\pi}_{t+12}) + \beta_2 e_t + u_t, \tag{4.2} \]

where \( \tilde{r}_t = r_t - \hat{\phi} y_t \), \( y_t \) is the (stationary) output gap and \( \hat{\phi} \) is the coefficient estimated through an auxiliary regression obtained by an OLS regression of the interest rate over the output gap and appropriate deterministic terms.

The resulting FM-OLS estimates of monetary policy coefficients in Equation (4.2) could be then interpreted as the central banks’ reaction functions to inflation gap and exchange rate movements \emph{conditional} on the state of the business cycle, as defined by the output gap. However, this method does not allow for estimating the slope parameter of the output gap. A useful way to recover this coefficient is to use the implied long-run coefficient from the error-correction regression used to test for linear cointegration. Indeed, as suggested by Pesaran et al. (2001), if the null hypothesis of no cointegration is rejected, the output gap coefficient can be properly interpreted as a long-run level relationship with the interest rate. Thus, conditional on the outcome from the bound testing approach, we report output gap coefficients implied from the error-correction regression.

### 4.1 Linear cointegration

Panel A of Table 2 reports the results of the Banerjee et al. (1998, BDM hereafter) and Pesaran et al. (2001, PSS hereafter) tests for linear cointegration. In particular, we rewrite Equation (4.2) by taking the following error-correction representation:

\[ \Delta r_t = \alpha + \lambda (r_{t-1} - \delta, t) + \beta (x_{t-1} - \delta, x_t) + \sum_{i=1}^{p-1} \phi_i \Delta z_{t-i} + \omega' \Delta x_t + u_t. \tag{4.3} \]

Equation (4.3) is an ARDL model of cointegration (see Hassler and Wolters, 2006, for a recent survey), with \( x_t = [(E_t \pi_{t+12} - \bar{\pi}_{t+12}), e_t, y_t]' \), \( z_t = [r_t, x_t]' \), and \( u_t \) is \( i.i.d \sim N(0, \sigma_u^2) \). Further, the deterministic trend is restricted to belong to the long-run relationship (see footnote 12 for more details). Both cointegration test procedures use the \( t \)-statistic asso-
ciated with parameter \( \lambda \) in Equation (4.3) to test the null hypothesis of no-cointegration \((\lambda = 0)\).\(^{11}\)

However, they differ slightly in terms of the testing approach. The BDM approach is a standard point test for the absence of a long-run relationship among the series. The PSS approach is instead a bound test where all regressors are either purely I(1) or I(0): if the estimated \( t \)-statistic falls below (above) the lower (upper) bound critical value, then a stationary long-run level relationship does (not) exist and conclusive inference can (not) be drawn. In addition to the \( t \)-statistic, the PSS test involves a Wald statistic to test jointly the null hypothesis of \( \lambda = 0 \) and \( \beta = 0 \).\(^{12}\)

[Table 2 about here]

The statistics described above have non-standard asymptotic distributions, which depend on the deterministic terms and the number of regressors in the model. Hence, we use unbiased exact \( p \)-values for the BDM statistic tabulated by Ericsson and MacKinnon (2002) and we compute exact small-sample 5% critical values for the PSS statistics through Monte Carlo simulations.

The test results show that the null hypothesis of absence of cointegration can be rejected in all countries. For Brazil and Chile, both the PSS \( t \)-statistic and Wald test statistic fall above the upper-bound 5% critical value, suggesting the presence of stationary long-run relationships among the variables. For Colombia and Mexico, the PSS Wald test statistic falls above the upper-bound 5% critical value, while using the same level of acceptance, the PSS \( t \)-statistic falls within the bounds. However, the BDM test does reject the null of no cointegration for all countries (slightly above the 5% level for Mexico), supporting the hypothesis of a level relationship among the full set of variables.

\(^{11}\)The main advantages of these error-correction-based methodologies are twofold (Ericsson and MacKinnon, 2002). First, the long-run coefficients (on which the hypothesis of cointegration is tested) are not biased, due to the inclusion of the short-run dynamics of the model into the test equation. Second, no restrictions are imposed on the long- and the short-run coefficients, given that the equilibrium and the dynamic relationships described by the model are estimated simultaneously. In addition, a desirable feature of PSS bounds testing approach is that the existence of long-run relationships among a set of covariates can be tested while being agnostic about the order of integration of the relevant variables. The test procedure is then applicable whenever the regressors are I(1), I(0) or mutually cointegrated.

\(^{12}\)The restriction over the deterministic trends is necessary in our testing strategy. Indeed, the PSS statistic we use for testing the presence of a long-run relationship sets the trend coefficient to zero under the null hypothesis. If the trend coefficient \( \delta \) was not subject to this restriction, Equation (4.3) would imply a quadratic trend in the level of policy interest rates under the null hypothesis of \( \lambda = 0 \) and \( \beta = 0 \), which is empirically implausible.
Because the presence of long-run relationships is not rejected by the data for all the countries under consideration, we can compute the output gap coefficients implied by the ECM (4.3). These coefficients (and their standard errors) are also reported in Table 2. Estimated coefficients are correctly signed and of reasonable magnitude in Brazil, Chile and Colombia, suggesting that central banks implemented monetary policy in a countercyclical manner over the period under analysis. By contrast, in the case of Mexico the output gap coefficient is not precisely estimated. de Mello and Moccero (2009) also report an insignificant coefficient for the output gap in Mexico for a monetary policy reaction function estimated in first differences.

Conditional on the output gap slope coefficient, Equation (4.2) is estimated by fully-modified OLS (Phillips and Hansen, 1990). The results are reported in Panel B of Table 2. Parameter estimates for the inflation gap are signed as expected and of reasonable magnitudes, indicating that central banks tighten monetary policy when inflation expectations rise above the central target. Parameter estimates for the exchange rate in Chile and Mexico are not statistically significant, while for Colombia the estimated coefficient is negatively signed, although extremely small in magnitude. This evidence is consistent with de Mello and Moccero (2011) and Schmidt-Hebbel and Werner (2002) for Brazil, Chile and Mexico. For Colombia, this result is mainly due to non-sterilized, balance-of-payment appreciating pressures on the exchange rate at a time when monetary policy was tightened to fight a rising inflation rate and to moderate a booming internal demand.  

4.2 Non-linear cointegration

Monetary reaction functions need not be linear, as suggested by the empirical literature for the United States (Dolado et al., 2004; Kim et al., 2005; Qin and Enders, 2008) and a few European countries (Bec et al., 2002; Bruinshoofd and Candelon, 2004; Taylor and Davradakis, 2006). Non-linearity may result from deviations from the conventional minimization of quadratic loss functions (Bec et al., 2002; Dolado et al., 2004; Cukierman and Muscatelli, 2008) and the presence of more complex price-setting mechanisms than those

---

13The appreciation of the exchange rate during the period under analysis is related to an improvement in terms-of-trade, an increase in net factor income from abroad and a rise in direct foreign investments (in oil extraction-related activities and the privatization of several state-owned companies). See Central Bank of Colombia (2007, 2008) for an analysis of recent exchange rate dynamics in Colombia.
subsumed in a linear Phillips curve (Nobay and Peel, 2003). For example, the central bank may react more strongly when the inflation lies above the target than when it is below it. Nominal wages can be flexible upwards but not downwards, leading to a convex, rather than linear, Phillips curve.

However, the empirical literature has dealt essentially with stationary data. To our knowledge, there are very few studies testing for cointegration and estimating cointegrated non-linear regressions (see, for instance, Christopoulos and León-Ledesma, 2007, on the long-run Fisher effect), and virtually no contributions extending this analysis to monetary reaction functions.

In what follows, we propose to shed additional light on the issues discussed in the previous section by testing for neglected non-linearity in the linear cointegrating equations and estimating non-linear reaction functions.

### 4.2.1 Testing for non-linearity in cointegrating equations

We use the Breitung (2001) rank procedure to test for the presence of neglected non-linearity in linear cointegrating equations. This procedure implies the inclusion in our cointegrating regressions of a function $f^*(E_t\pi_{t+12} - \bar{\pi}_{t+12}, e_t)$ describing the non-linear part of the model. Although the actual specification of $f^*(\cdot)$ is unknown, it can be approximated by Fourier series and neural networks (Granger, 1995). To some extent, this approximation can be obtained through a rank transformation ($R_T$) of non-stationary regressors $(E_t\pi_{t+12} - \bar{\pi}_{t+12}, e_t)$, and it is related to the neural network approach advocated in Lee et al. (1993). Given a vector $x_t$ of input variables for the non-linear term, the neural network approach approximates $f^*(x_t)$ by $\sum_{j=1}^q \phi_j \psi(x_t a_j)$, which can be simplified as $\phi \psi(x_t a)$ when $\psi(\cdot)$ is a bounded function, and $x_t$ is a vector of scalar variables. The use of the rank transformation can be motivated by letting $T^{-1}R_T(x_t) = F_T(x_t) = \psi(x_t a)$ be the empirical distribution function, where parameter $a$ can be dropped due to the invariance of the rank transformation. Therefore, Breitung (2001) proposes a test for neglected non-linearity involving the multiple of the rank transformation $\theta \cdot R_T(x_t)$ instead of $f^*(x_t)$ itself, where $\theta$ is a parameter and $R_T$ is a rank function.

The null hypothesis of linear against non-linear cointegration is then tested in three steps. First, following Breitung (2001), a linear cointegrating equation is estimated by ef-
ficient DOLS regressions (Stock and Watson, 1993). Second, the residuals from the linear cointegrating equation ($\varepsilon_t$) are regressed on the same regressors and the rank transformation $R_T(E_t\pi_{t+12} - \bar{\pi}_{t+12}, e_t)$, therefore embedding the alternative hypothesis of neglected non-linearity. Finally, the score test statistic for the significance of $R_T(E_t\pi_{t+12} - \bar{\pi}_{t+12}, e_t)$ is computed as follows:

$$T \cdot R^2 = \sigma^2 \hat{\beta} [R_T(x_t)'R_T(x_t) - R_T(x_t)'x_t(x_t'x_t)^{-1}x_t'R_T(x_t)],$$

where $R_T(x_t) = R_T(E_t\pi_{t+12} - \bar{\pi}_{t+12}, e_t)$, $x_t = [E_t\pi_{t+12} - \bar{\pi}_{t+12}, e_t]$, $\sigma^2 = \varepsilon_t'\varepsilon_t/T$, and $\hat{\beta}$ is the least squares estimate of the vector of rank transformation parameters. Breitung (2001) shows that this score test is distributed asymptotically as $\chi^2$ with $k$ degrees of freedom (where $k$ is the number of integrated regressors) under the null hypothesis of linearity.

The results of the rank tests are reported in Table 3. The hypothesis of linear cointegration can be rejected for Brazil, Colombia and Mexico, although only at the 10% significance level in some cases. On the other hand, the hypothesis of linearity cannot be clearly rejected for Chile at standard levels of significance. On the basis of these findings, the linear cointegration framework appears to be more suitable for the estimation of monetary reaction functions in the case of Chile, but not for the other countries in the sample.

4.2.2 Estimating the non-linear cointegrating equations

Non-linear cointegrating equations were estimated for Brazil, Colombia and Mexico as smooth-transition regression (STR) processes, defined as follows:

$$\tilde{r}_t = \alpha + \delta_t t + \beta_{1,1}(E_t\pi_{t+12} - \bar{\pi}_{t+12}) + \beta_{1,2} e_t$$
$$+ (\alpha_2 + \delta_2 t + \beta_{2,1}(E_t\pi_{t+12} - \bar{\pi}_{t+12}) + \beta_{2,2} e_t)G(\gamma, c, E_t\pi_{t+12} - \bar{\pi}_{t+12}) + u_t,$$
which is a non-linear equivalent of Equation (4.2) and where

\[ G(\gamma, c, \pi_{t+12} - \pi_{t+12}) = \left(1 - \exp\left(-\frac{\gamma}{\sigma^2(E_t\pi_{t+12} - \pi_{t+12})}ight) \times [(E_t\pi_{t+12} - \pi_{t+12}) - c]^2 \right). \]

As anticipated in Section 4, in order to deal with both stationary and non-stationary variables in our non-linear cointegrating framework, we exploit the asymptotic properties of the NLLS estimator discussed by Chang et al. (2001) and follow their proposed methodology. As in the case of linear cointegrated regressions (Park and Phillips, 1989), the limiting distribution of estimated coefficients of stationary regressors should not be affected by the presence of non-stationary variables. Due to this asymptotic orthogonality, we run a first-step OLS regression involving the dependent variable \( r_t \) and the stationary regressor \( y_t \), use the estimated coefficients to remove the linear effect of the stationary regressor on the regressand \( \tilde{r}_t \), and then proceed with the non-linear estimation (see Chang et al., 2001, p. 15, for more details).

We focus on the exponential specification of the transition function, because in an inflation zone targeting setting, policy responses are hypothesized to be moderate when inflation lies within the target range and to become more forceful as inflation approaches the boundaries of the target zone. Accordingly, the inflation gap is the variable describing transition across policy regimes. The slope parameter \( \gamma > 0 \), which defines the degree of smoothness of the transition function, \( G(\gamma, c, E_t\pi_{t+12} - \bar{\pi}_{t+12}) \), is normalized by the variance of the transition variable (inflation gap), as suggested by Granger and Teräsvirta (1993). Parameter \( c \) is the threshold location parameter, implying that there are two policy regimes. Furthermore, if \( \gamma \to 0 \), then \( G(\cdot) \to 0 \) and the reaction function becomes

\[ \tilde{r}_t = \alpha_1 + \delta_1t + \beta_{1,1}(E_t\pi_{t+12} - \bar{\pi}_{t+12}) + \beta_{1,2}e_t. \]

By contrast, when \( \gamma \to +\infty \), the reaction function collapses into a threshold model.\(^{14}\) For intermediate values of \( \gamma \), the monetary response switches smoothly from one regime to another. Finally, the transition function is a bounded (between 0 and 1) continuous function of the transition variable.

There is a large literature on the asymptotic properties and estimating procedures for broad classes of non-linear models (e.g., Granger and Teräsvirta, 1993, and Teräsvirta,

\(^{14}\)When \( \gamma \to +\infty \), the reaction function becomes \( \tilde{r}_t = \alpha_1 + \delta_1t + \beta_{1,1}(E_t\pi_{t+12} - \bar{\pi}_{t+12}) + \beta_{1,2}e_t \), if \( E_t\pi_{t+12} - \bar{\pi}_{t+12} = c \), and \( \tilde{r}_t = (\alpha_1 + \alpha_2) + (\delta_1 + \delta_2)t + (\beta_{1,1} + \beta_{2,1})(E_t\pi_{t+12} - \bar{\pi}_{t+12}) + (\beta_{1,2} + \beta_{2,2})e_t \), if \( E_t\pi_{t+12} - \bar{\pi}_{t+12} \neq c \).
such as those described by Equation (4.5). In particular, we are interested in a narrower class of non-linear regressions that includes integrated series. An asymptotic theory for such models is developed in the seminal work of Park and Phillips (2001), where the authors prove the consistency of the non-linear least squares estimator (NLLS), under some regularity conditions.

A further development of the asymptotic theory discussed in Park and Phillips (2001) can be found in Chang and Park (2003), who consider non-linear index models driven by integrated time series. They show that smooth transition regressions can be considered as a special case in their framework, representing a long-run cointegrating relationship that departs from a long-run equilibrium and smoothly adjusts to a new equilibrium (see the Appendix for more details). The assumptions advocated by Chang and Park (2003) rule out multicointegration and define some boundary and differentiation conditions on the STR function $G(\cdot)$, but involve a somewhat restrictive multivariate invariance principle (Phillips and Durlauf, 1986) adopting martingale difference errors rather than a more general error process. This restriction is apparently constraining on the limit boundary condition of $G(\cdot)$ ($\lim_{x \to -\infty} G(x) = 0$ and $\lim_{x \to \infty} G(x) = 1$), but a) any smooth bounded function with well defined asymptotes can be normalized so that it satisfies this condition, and b) the limiting distributions of estimated parameters do not depend upon $G(\cdot)$, implying that the estimators are consistent even if the STR function is misspecified (Chang and Park, 2003, p. 77 and 83).\(^{15}\)

We use the Marquardt-Levenberg optimization algorithm to implement the NLLS estimator. Since many parameters have to be estimated, we proceed with a simple identification strategy. Because the parameters of the transition function are the most difficult to estimate, we first concentrate on the linear and non-linear parameters, fixing $\gamma$ equal

\(^{15}\)Assumption b) is criticized by Saikkonen and Choi (2004) and Choi and Saikkonen (2010), who develop a NLLS asymptotic theory based on the triangular array asymptotics. Such asymptotic theory exhibits suitable limiting properties for the case of smooth transition parameters, and seems to overperform the limiting theory exposed in Chang and Park (2003). Moreover, simulation results in Saikkonen and Choi (2004) suggest that a Gauss-Newton estimator performs better than a simple NLLS estimator, mainly in an efficient lead-and-lags regression problem. However, for reasons discussed in footnote (18), we do not implement such estimator.
to 1 and \( c \) equal to 0 to obtain a set of initial values for those parameters.\(^{16}\) Then, in a second round of estimations, the restriction on the constant \( c \) is relaxed, and a grid search is implemented for the optimal initial value of \( c \) using an interval delimited by the minimum and maximum values of the transition variable. These estimated parameters are in turn used as initial values in the third step of the estimation, where the restriction on the slope parameter \( \gamma \) is relaxed, and a grid search is performed again to obtain the optimal initial values for \( \gamma \) within an interval from 1 to 10. Finally, we compute initial values for all relevant parameters and implement the estimation procedure for the last time on the basis of these new initial values. Insignificant parameters are discarded, and a new estimation loop is run until a final model is obtained.

The asymptotic properties of the NLLS estimator are valid as long as all integrated regressors are exogenous.\(^{17}\) Chang et al. (2001) and Chang and Park (2003) extend the theory developed in Park and Phillips (2001) and propose a simple methodology to deal with endogeneity in a non-linear cointegrating regression, which is closely related to the fully-modified regression model (Phillips and Hansen, 1990) and the canonical cointegration regression procedure (Park, 1992). We follow this approach and compute an efficient non-stationary non-linear least squares estimator (EN-NLLS), which corrects for the correlation between the residuals of the long-run regression and innovations in the integrated regressors. It is important to deal with endogeneity in the present case, because expected inflation and the exchange rate can also react to interest rate changes.\(^{18}\)

A few steps are necessary to compute the EN-NLLS estimator. We first compute the fitted residuals \( \hat{u}_t \) from the estimated baseline non-linear model (4.5) and then estimate

\(^{16}\) As mentioned before, while \( \gamma \) is a free parameter, restricted to be greater than zero, \( c \) must lie between the minimum and the maximum values of the transition variable to be economically interpretable. We therefore fix \( \gamma \) equal to 1 and \( c \) equal to 0, which lies between the maximum and the minimum values of the inflation gap for Brazil and is very close to the minimum value for Colombia and Mexico, as initial values for the NLLS estimations.

\(^{17}\) Park and Phillips (2001) show that least-square regressions are consistent even when the model is non-linear, but the rates of convergence can differ from the case of regressions with stationary data. Also, in a multivariate setting the asymptotic distribution of the NLLS estimator is in general non-Gaussian, which implies that standard hypothesis testing is invalid. Only in the special case where the integrated regressors are strictly exogenous is the asymptotic distribution of the NLLS estimator mixed-normal.

\(^{18}\) Another theoretically efficient estimator consists of including leads and lags of the first-differenced regressors, as suggested by Saikkonen and Choi (2004) and Choi and Saikkonen (2010). We experimented with the leads-and-lags estimator, using one lead and lag for each non-stationary regressor, and the results (not reported) are consistent with those obtained on the basis of the EN-NLLS estimator. However, it was difficult to estimate the regressions with more than one lead and lag because of the fall in the degrees of freedom resulting from the inclusion of additional parameters.
the following regression:

\[ v_t = \hat{\Pi}_1 v_{t-1} + \cdots + \hat{\Pi}_\ell v_{t-\ell} + \hat{\eta}_{\ell,t}, \]

where \( v_t = \Delta(\pi_{t+12} - \bar{\pi}_{t+12}, e_t)' \) is the vector of innovations to the integrated variables, and the autoregressive order \( \ell \) increases as \( T \to \infty \). For the choice of this parameter, we follow Chang et al. (2001) and we let \( \ell = T^d \), where \( d \) is chosen such that \( 0 < d < 1/8 \).

We subsequently retrieve the one-period-ahead fitted residuals \( \hat{\eta}_{\ell,t+1} \) to compute:

\[ \hat{\sigma}_{\eta} = \frac{1}{T} \sum_{t=1}^{T} \hat{u}_t \hat{\eta}_{\ell,t+1}' \quad \text{and} \quad \hat{\Sigma}_{\eta\eta} = \frac{1}{T} \sum_{t=1}^{T} \hat{\eta}_{\ell,t} \hat{\eta}_{\ell,t}'. \]

Finally, we define a modified dependent variable:

\[ \tilde{r}_t^* = \tilde{r}_t - \hat{\sigma}_{\eta}\hat{\Sigma}_{\eta\eta}^{-1}\hat{\eta}_{\ell,t+1}, \]

and re-estimate the non-linear models following the identification strategy described above:

\[ \tilde{r}_t^* = \alpha_1 + \beta_{1,1}(E_t \pi_{t+12} - \bar{\pi}_{t+12}) + \beta_{1,2} e_t + (\alpha_2 + \beta_{2,1}(E_t \pi_{t+12} - \bar{\pi}_{t+12}) + \beta_{2,2} e_t)G(\gamma, c, E_t \pi_{t+12} - \bar{\pi}_{t+12}) + u_t^*, \quad (4.6) \]

where \( u_t^* = u_t - \hat{\sigma}_{\eta}\hat{\Sigma}_{\eta\eta}^{-1}\hat{\eta}_{\ell,t+1} \). The asymptotic behavior of the EN-NLLS estimator is then closely related to the limit theory for the NLLS estimator and also efficient in the sense of Phillips (1991) and Saikkonen (1991).

### 4.2.3 Checking for non-linear cointegration

A final issue arises when checking for the stationarity of the residuals of the non-linear cointegrating regression. In a recent paper, Choi and Saikkonen (2010) propose a test statistic based on the residuals of the non-linear equations, where non-linear cointegration is the null hypothesis against an alternative of no cointegration. However, tabulations of this statistic are impractical due to non-linearity, a problem that cannot be solved through bootstrapping or sub-sampling methods. The authors therefore propose a new

\[ \text{The procedure is akin to the Shin (1994) test for the null of cointegration, following the tradition of the Kwiatkowski et al. (1992) univariate test for the null of stationarity.} \]
test statistic using sub-residuals and the Bonferroni procedure, defined as:

\[ C_{\text{NLLS}}^{b,i} = b^{-2} \hat{\omega}_u^{-2} \sum_{t=i}^{i+b-1} \left( \sum_{j=i}^{t} \hat{u}_j \right)^2, \]  \hfill (4.7)

\[ C_{\text{EN-NLLS}}^{b,i} = b^{-2} \hat{\omega}_{u^*}^{-2} \sum_{t=i}^{i+b-1} \left( \sum_{j=i}^{t} \hat{u}_{j}^* \right)^2, \]  \hfill (4.8)

where \( \hat{\omega}_u \) and \( \hat{\omega}_{u^*} \) are consistent estimates of the long-run variance of the residuals from Equation (4.6) and (4.5), respectively, \( b \) is the block size of the sub-residuals, and “\( i \)” denotes the starting point of sub-residuals \( \{ u_t \}_{t=i}^{i+b-1} \) and \( \{ u_{t}^* \}_{t=i}^{i+b-1} \). Choi and Saikkonen (2010) show that, when \( b \to \infty \) and \( b/T \to 0 \) as \( T \to \infty \), the statistic is asymptotically distributed as:

\[ C_i \to \int_0^1 W^2(s)ds, \]  \hfill (4.9)

where \( \iota = \{ \text{NLLS}, \text{EN-NLLS} \} \) and \( W \) is a standard Brownian motion. Finally, the test statistics is computed as:

\[ C_{\iota}^{b,\text{max}} = \max(C_{\iota}^{b,i_1}, \ldots, C_{\iota}^{b,i_M}), \]  \hfill (4.10)

where \( i_1, \ldots, i_M \) is the number of starting points of sub-residuals and \( M \) is the number of sub-residuals-based tests. Combining (4.10) with the asymptotic result in Equation (4.9) and the Bonferroni inequality yields:

\[ \lim_{T \to \infty} P[C_{\iota}^{b,\text{max}} \leq c_{\alpha/M}] \geq 1 - \alpha, \]  \hfill (4.11)

which implies that \( \alpha \)-level critical values for the test statistic are taken from the distribution of \( \int_0^1 W^2(s)ds \) using the level \( \frac{\alpha}{M} \).\(^{20}\)

\(^{20}\)The cumulative distribution function of \( \int_0^1 W^2(s)ds \) takes the following form:

\[ \text{cdf}(z) = \sqrt{2} \sum_{n=0}^{\infty} \frac{\Gamma(n + 1/2)}{\sqrt{\pi}n!\Gamma(1/2)} (-1)^n \left[ 1 - \text{Erf} \left( \frac{u}{2\sqrt{z}} \right) \right], \]

where \( u = (\sqrt{2}/2) + 2n\sqrt{z}, \) \( \text{Erf}(u) = (2/\sqrt{\pi}) \int_0^u \exp(-\xi^2)d\xi \) is the error function of a gaussian distribution and the series is truncated at small \( n \) (we follow Choi and Saikkonen, 2010 and set \( n = 10 \)).
For a given block size $b$, the authors propose a simple rule to choose the optimal number of starting points of the sub-residuals ($i_j$) and the number of tests ($M$) used in the Bonferroni procedure. In addition, the selection of the block size $b$ can be carried out by using either a fixed rule, such as by fixing $b = [T^{0.7}]$, with $[z]$ denoting the integer part of $z$, or a minimum volatility rule. In the latter case, $b$ should be chosen to minimize the standard deviation of the test statistics for each value of $b$ from $b_i = b_{small}$ to $b_i = b_{big}$, with $i = (small + m, \ldots, big - m)$, $b_{small} = [T^{0.7}]$ and $b_{big} = [T^{0.9}]$, and $m = 2$ (Romano and Wolf, 2001; Choi and Saikkonen, 2010).

4.2.4 The parameter estimates

The results of the non-linear cointegration models are reported in Table 4. The parameters of interest are signed as expected and of reasonable magnitudes. The NLLS and EN-NLLS estimators produce comparable estimates, suggesting that endogeneity may not be a major problem in these estimations.

[Table 4 about here]

In the case of Brazil, the estimated coefficient on the inflation gap is statistically significant and positively signed in the linear part of the model (around 0.9, according to EN-NLLS estimates), while being insignificant in the non-linear part. This result suggests that the Central Bank of Brazil responds linearly to deviations of expected inflation from the target, irrespective of the size of the expected inflation gap. In other words, the central bank’s response to the inflation gap does not change across regimes. In contrast, central bank responses to the exchange rate appear to vary across policy regimes in Brazil. The net effect is positive (around 1.2, for the EN-NLLS estimates), suggesting that the central bank reacts by hiking the interest rate when the exchange rate depreciates. But this response loses vigour as the inflation gap widens (Figure 4b). In other words, the central bank’s response to the exchange rate seems to be stronger when the inflation gap is close to the threshold. The theoretical literature offers no guidance on how the monetary authorities should respond non-linearly to the exchange rate dynamics. Nevertheless, our empirical finding suggests that under inflation zone targeting the monetary authorities
may focus on exchange rate management only in so far as inflation is within the target zone.

We attribute the weaker response by the monetary authorities as inflation deviates from the target to the sequence of adverse supply shocks that hit the Brazilian economy in the early phase of inflation targeting. These shocks, such as a severe energy shortage in 2001 and a confidence crisis in the run-up to the presidential election of October 2002, when commitment by the front-running candidate to macroeconomic austerity was in doubt, resulted in a sizeable exchange rate depreciation and an attendant impact on inflation and inflation expectations. Cognizant that monetary action under such circumstances would be overly destabilizing, the monetary authority opted for pursuing adjusted targets while committing to tackling the second-round effects of exchange rate devaluations on inflation.

Two final observations are noteworthy in the case of Brazil. First, the estimated threshold is negative, as noted above, albeit quite small in magnitude (around -0.3). We interpret this finding as reflecting the fact that the central bank may have attempted to consolidate credibility in the monetary policy regime by seeking to anchor inflation expectations somewhat below the central target. Second, the slope parameter, which reflects the smoothness of the transition function, is relatively low in magnitude (around 3.9, on the basis of EN-NLLS estimates). This implies that regime shifts are fairly slow, once the transition variable has reached the estimated threshold. This is illustrated by the transition function and regime changes depicted in Figures 2a and 3a.

The estimated transition function can be used to assess the behavior of the central bank as the inflation gap approaches the boundaries of the inflation target band (Figure 2a). In Brazil, the central bank operates in the “lower-response” regime, where the value of the transition function is 1, when expected inflation exceeds the central target by about 1 percentage points or falls short of it by about 1.5 percentage points. This suggests a policy regime shift that takes place within a range of values for the inflation gap that is much narrower than the formal width of the tolerance band around the central target.

21The significance of this parameter, although reported in Table 4, is hardly interpretable, because we cannot use conventional hypothesis testing for the parameters inducing non-linearity due to identification issues (Saikkonen and Choi, 2004).
which is currently plus or minus 2 percentage points. Also, central bank responses to the exchange rate have strengthened since March 2005, because the transition function has barely reached the value of 1, when the “low-response” regime dominates, since then and its values have been lower on average. A progressive loosening can nevertheless be observed in the latter part of the sample (Figure 3a) as a possible reaction to the stabilization of the Brazilian currency after the devaluation shock of 2008-2009 following large capital outflows.

With regard to Colombia, the inflation gap coefficient is statistically significant and positively signed in the linear part of the model, and negatively signed in the non-linear part. In this case, as opposed to Brazil, the central bank’s response to the inflation gap appears to change across regimes. For example, when expected inflation is close to the target (the threshold for regime change is estimated to be 0.3 in this case, see below), the transition function tends to 0, and the linear part of the model dominates. However, non-linear responses emerge when expected inflation deviates from the target, and the transition function tends to 1 (Figure 4c). The net effect, calculated as the sum of the coefficients estimated for both regimes cancels out (we cannot reject the null hypothesis of equality of these coefficients tested through a Wald test); as a result, only the constrained coefficients are reported.\textsuperscript{22} In other words, the central bank reacts to an increase (decrease) in the inflation gap by tightening (loosening) monetary policy, but this policy response appears to weaken as the inflation gap widens.\textsuperscript{23}

As for the exchange rate in Colombia, policy responses are positive in the linear part of the model and negative in the non-linear part. As opposed to the linear findings, we have then evidence that monetary policy is tightened in response to exchange rate depreciations. However, non-linear estimates point out some episodes of loosening of the central bank responses, which is consistent with an uneven pattern of trending appreciation of the colombian peso during the last decade. Indeed, when the transition function reaches the value of 1, the net effect across regimes is negative although extremely small. Further,

\textsuperscript{22}It follows from Theorems 4.1 and 5.1 in Park and Phillips (2001) and from Chang et al. (2001) that standard hypothesis testing, such as Wald tests, is valid for parameters in both linear and non-linear part of the model.

\textsuperscript{23}Fraga et al. (2003) present evidence suggesting that inflation targeting emerging-market economies perform less well than developed economies because inflation targeting is more challenging in the former than in the latter, rather than because of lack of commitment.
from Figure 4d it appears that the monetary authority has not responded on average to exchange rate movements. The threshold parameter is around 0.3, suggesting that the central bank reacts to deviations of inflation expectations from slightly above the target. The slope parameter is found to be relatively low in magnitude (around 3, on the basis of EN-NLLS estimates), suggesting that regime shifts are fairly slow, almost as much as in the case of Brazil. The estimated transition function and regime changes are depicted in Figures 2b and 3b, respectively.\(^{24}\) In particular, the increase in the value of the transition function between 2005 and 2010 suggests that the authorities may have been increasingly reluctant to use monetary policy as an exchange-rate management tool, while letting the Colombian peso to appreciate on the back of the factors mentioned before (Figure 2b).

The EN-NLLS estimation results for Mexico show that the inflation gap is statistically different from zero both in the linear and non-linear parts of the model. As in Colombia, the Mexican monetary authority reacts strongly to the inflation gap when it is close to the threshold of about 0.5 percentage point. However, sizeable inflation gaps prompt a weaker response by the central bank. Unlike the case of Colombia though, the Mexican central bank reacts to inflation expectations even when they deviate substantially from the target, because the sum of the coefficients estimated for both regimes is strictly positive.\(^{25}\) Finally, the estimated transition threshold is positive (as mentioned before), and the smoothness parameter is quite high in magnitude, suggesting that regime shifts are fairly swift.

The estimated transition function and regime changes are depicted in Figures 2c and 3c, respectively. A shift to the “lower response” regime takes place when expected inflation exceeds the central target by up to 0.3 percentage point. There is also evidence of low-response behavior as expected inflation exceeds the central target by about 0.7 percentage point. This asymmetry may reflect the central bank’s evaluation of risks associated with deviations of inflation expectations from the central target. The central bank may evaluate the deflationary risks related to small negative inflation gaps, which call for monetary

\(^{24}\)Although the magnitude of the smoothness parameter is similar in both countries, the transition functions for Colombia and Brazil are different because the scale factor \(\sigma^2_{\pi_t - \bar{\pi}}\) is larger for Brazil.

\(^{25}\)The monetary policy response coefficients to the inflation gap may look high in the cases of Colombia and Mexico. However, it should be reminded that coefficients depend on the scale of the dependent and explanatory variables. In Colombia, the average policy interest rate over the estimation period amounts to close to 7%, while the average of the expected inflation gap is low, at close to 0.45%. Hence, the difference in scale between the two variables results in a high response coefficient in the monetary policy reaction function. A similar reasoning can be applied to the case of Mexico.
loosening, to be lower than the inflationary risks associated with small positive inflation gaps, which call for a tightening of the policy stance. However, the monetary authorities reacted particularly aggressively to the inflation gap on few occasions, namely in 2002 and between 2006 and 2007 (Figure 4e).

[Table 5 about here]

We finally test for stationarity of the residuals of the non-linear reaction functions to make sure that the variables of interest cointegrate. For this purpose, we follow the procedure developed by Choi and Saikkonen (2010) and described above. The results of the non-linear cointegration tests over both NLLS and EN-NLLS estimates are reported in Table 5. For ease of exposition, we only report the test results for the EN-NLLS estimations with ℓ = 1. The Quadratic Spectral kernel is used to estimate the long-run variance of residuals, along with either an automatic or a fixed bandwidth selection method (Andrews, 1991; Andrews and Monahan, 1992; Kwiatkowski et al., 1992).

The test statistics show that the null hypothesis of non-linear cointegration cannot be rejected at the adjusted 5% level for all countries, regardless of whether the fixed or the minimum volatility rules are used. Despite marginal evidence of rejection for Brazil and Mexico (NLLS and EN-NLLS residuals, fixed rule and automatic bandwidth), the test results strongly suggest the existence of a stationary relationship in our non-linear monetary reaction functions for all the countries under analysis.

5 Conclusions

This paper tested empirically the inflation zone targeting theory of Orphanides and Wieland (2000) for the case of four Latin American countries: Brazil, Chile, Colombia and Mexico. The theory predicts that the optimal monetary policy rule for inflation zone targeters is to respond moderately to inflation when it lies within a permissible range and increasingly strongly as inflation deviates (symmetrically) from the target range.

We tested the theory by estimating unrestricted monetary policy reaction functions for the four countries using monthly data for the post-1999 inflation-targeting period. We tested for cointegration among the inflation gap, the interest rate and the exchange rate,
due to the presence of unit roots in the series, conditional on the output gap. Strong evidence of neglected non-linearity was found in the linear cointegrating equations for Brazil, Colombia and Mexico on the basis of the Breitung (2001) test, as well as of non-linear cointegration in the data for these three countries using the Choi and Saikkonen (2010) test. Smooth-transition models with an exponential transition function, where the transition variable is the inflation gap, were estimated using the non-linear least squares (NLLS) and the efficient non-stationary non-linear least squares (EN-NLLS) methodologies developed by Park and Phillips (2001) and Chang et al. (2001). In the case of Chile, only linear cointegration was tested for, given that no evidence of neglected non-linearity could be found.

The results reported above suggest that central banks react to increases in the inflation gap by tightening monetary policy. Central bank behavior is linear in Brazil, because the estimated policy response is statistically insignificant in the non-linear part of the model, and in Chile, where statistical tests rejected non-linearity. Evidence of non-linear monetary responses was found for Colombia and Mexico. In both countries, a negative coefficient on the inflation gap in the non-linear part of the model suggests that policy responses weaken when the inflation gap widens, a finding that is against the inflation zone targeting literature but can be attributed to the sequence of adverse supply shocks that have hit these economies, rather than to a lack of resolve on the part of the monetary authorities to act decisively when confronted with large inflation gaps.

Our findings are in line with a large literature showing that, in the context of emerging-market economies, greater monetary policy flexibility is required to deal with exogenous shocks, which are typically stronger than in the case of developed nations. Exchange rate changes have a longer and stronger impact on inflation and the real economy in emerging-market economies than in developed economies, due to more sizeable terms-of-trade swings and vulnerability to “sudden stops” of capital inflows. Because such factors are not accounted for in inflation zone targeting theory, the monetary policy response will be different. Ball (1999) suggests that in small open economies, where exchange-rate changes strongly affect inflation outcomes through import prices, the monetary authorities should target “long-run” inflation (i.e., inflation that is not influenced by the exchange-rate-to-import-price channel). Otherwise, targeting actual inflation would result in higher
output volatility. Fraga et al. (2003) suggest neutralizing second-order effects of supply shocks while accommodating first-round effects, an approach that was followed by the Central Bank of Brazil during the period 2003-2004. Another policy option consists of embedding explicit escape clauses or caveats in the monetary policy framework. For example, some inflation targeting countries, including New Zealand, Iceland and the Czech Republic allow for deviations of inflation from the target when the economy is hit by supply-side shocks (Kahn, 2009). In all circumstances, it is at least crucial that the central bank’s response be clearly explained to the general public, so as not to undermine central bank credibility.
References


Appendix: Consistency of NLLS and EN-NLLS estimators

Consider the NLLS estimate of the smooth-transition regression (4.5):

\[ Q_T(\theta) = \frac{1}{2} \sum_{t=1}^{T} (w_t - F(x_t, \theta))^2, \quad (A-1) \]

where \( F(x_t, \theta) \) is the right-hand side of Equation (4.5), \( x_t \) is the vector of non-stationary regressors, and \( \theta \) is the vector of parameters with true value \( \theta_0 \). The NLLS estimator \( \hat{\theta}_T \) of \( \theta \) is then:

\[ \hat{\theta}_T = \arg\min_{\theta \in \Theta} Q_T(\theta), \quad (A-2) \]

where \( \Theta \) is the parameter space, which is assumed to be compact and convex in \( \mathbb{R}^p \), and the residuals \( \hat{u}_t = w_t - F(x_t, \hat{\theta}_T) \) have estimated variance \( \sigma^2_{\hat{u}} = (1/T) \sum_{t=1}^{T} \hat{u}_t^2 \). Define \( \dot{Q}_T = \partial Q_T / \partial \theta, \ddot{Q}_T = \partial^2 Q_T / \partial \theta \partial \theta' \), \( \dot{F} = \partial F / \partial \theta \) and \( \ddot{F} = \partial^2 F / \partial \theta \partial \theta' \), so that:

\[ \dot{Q}_T(\theta) = - \sum_{t=1}^{T} \dot{F}(x_t, \theta)(w_t - F(x_t, \theta)), \]

\[ \ddot{Q}_T(\theta) = \sum_{t=1}^{T} \ddot{F}(x_t, \theta)\dot{F}(x_t, \theta)' - \sum_{t=1}^{T} \dddot{F}(x_t, \theta)(w_t - F(x_t, \theta)), \]

and the first-order Taylor expansion:

\[ \dot{Q}_T(\hat{\theta}_T) = \dot{Q}_T(\theta_0) + \ddot{Q}_T(\theta_T)(\hat{\theta}_T - \theta_0), \quad (A-3) \]

where \( \theta_0 \) is an interior point of \( \Theta \), and \( \theta_T \) is on the line segment joining \( \hat{\theta}_T \) and \( \theta_0 \). The limiting distribution of \( \hat{\theta}_T \) can be derived from Equation (A-3) as in the standard non-linear regression. Conditional on a set of assumptions about residuals, integrated processes and the transition function \( G(\cdot) \) (see Section 4.2.2 and footnote 15), the application of Lemmas 1 and 2 in Chang and Park (2003, p. 78-79) implies that:

\[ C_T^{-1} J' \dot{Q}_T(\theta_0) J C_T^{-1} \rightarrow_d A > 0 \quad \text{a.s.,} \quad (A-4) \]

\(^{26}\)Since the objective function is continuous on \( \Theta \) and the parameter space is compact and convex, the NLLS estimator of parameters exists and is Borel measurable (Pötscher and Prucha, 1997).
where

\[ A = \text{diag} \left( \int_0^1 N(r)N'(r) dr, \int_{-\infty}^\infty ds \int_0^1 dL_1(r,0)M(r,s)M(r,s)' \right), \]

and

\[-C_T^{-1}J'\dot{Q}_T(\theta_0) \rightarrow_d B, \quad (A-5)\]

where

\[ B = \left( \int_0^1 N(r)dU(r), \left( \int_{-\infty}^\infty ds \int_0^1 dL_1(r,0)M(r,s)M(r,s)' \right)^{1/2} W(1) \right). \]

Further details on the notation employed here (\(N, U, L_1, M, W\)) can be found in Park and Phillips (2001) and Chang and Park (2003). However, it is worth noticing that \(W\) is a vector Brownian motion with variance \(\sigma_u^2 I_m\). Consider a suitable normalizing sequence \(C_T\) of symmetric matrices and an orthogonal matrix \(J\). Chang and Park (2003) then show that:

\[ C_TJ'(\hat{\theta}_T - \theta_0) = -(C_T^{-1}J'\ddot{Q}_T(\theta_0)JC_T^{-1})^{-1}C_T^{-1}J'\dot{Q}_T(\theta_0) + \text{Op}(1) \rightarrow_d A^{-1}B, \quad (A-6)\]

and, if \(C_{T\delta} = T^{-\delta}C_T\) (for \(\delta > 0\)) and \(\Theta_T = \{\theta : ||C_{T\delta}(\theta - \theta_0)|| \leq 1\} \subset \Theta\), then:

\[ ||C_{T\delta}^{-1}J'(\ddot{Q}_T(\theta) - \ddot{Q}_T(\theta_0))JC_{T\delta}^{-1}|| \rightarrow_p 0, \quad (A-7)\]

which defines convergence in the probability of the NLLS estimator and ensures the asymptotics in Equation (A-6), provided the convergence in distribution in Equations (A-4) and (A-5) and the existence of \(C_{T\delta}\) (Wooldridge, 1994; Park and Phillips, 2001).

If the set of assumptions holds, then the parameters in Equation (4.5) are consistently estimated by NLLS when \(T \rightarrow \infty\), with a rate of convergence that varies across parameters. In particular, Equation (A-6) holds for \(C_T = \text{diag}(T^{1/2}, T^m, T^m, T^{3/4}, D_T)\) and \(J = \text{diag}(1, H, H, 1, H)\), where \(I_m\) is an identity matrix for the \(m\)-dimensional I(1) processes, \(D_T = \text{diag}(T^{3/4}, T^{5/4}I_{m-1})\) and \(H = (h_1, H_2)\) is an \((m \times m)\)-orthogonal matrix.
The asymptotic results reported above also hold for EN-NLLS regressions of smooth-transition models. Indeed, consider the NLLS estimate of the following non-linear regression:

\[ Q_T(\theta) = \frac{1}{2} \sum_{t=1}^{T} (w_t^* - F(x_t, \theta))^2, \quad (A-8) \]

where \( w^* \) is the dependent variable modified according to the efficient method discussed in Chang et al. (2001). Chang and Park (2003) show that

\[ C_T J'(\hat{\theta}_T - \theta_0) \rightarrow_d A^{-1}B^*, \quad (A-9) \]

and the convergence in probability stated in Equation (A-7) also applies. The EN-NLLS estimator \( \hat{\theta}_T^* \) of \( \theta \) has a mixed normal limiting distribution with (conditional long-run) variance \( \sigma^2_* = \sigma_u^2 - \omega_u\Omega^{-1}\omega_{vu} \). The EN-NLLS estimator is thus efficient in the sense of Phillips (1991) and Saikkonen (1991).
Figure 1: Interest rates, expected inflation and exchange rates

Source: Data available from the central banks of Brazil, Chile, Colombia and Mexico
Figure 2: Transition functions

(a) Brazil

(b) Colombia

(c) Mexico

Notes: Figures are generated using EN-NLLS estimates with $\ell = 1$. 
Figure 3: Regime changes

(a) Brazil

(b) Colombia

(c) Mexico

Notes: See Figure 2.
Figure 4: Central banks reactions with respect to inflation gap and exchange rate

(a) Brazil - inflation gap  
(b) Brazil - exchange rate  
(c) Colombia - inflation gap  
(d) Colombia - exchange rate  
(e) Mexico - inflation gap

Notes: See Figure 2.
Table 1: unit root tests

<table>
<thead>
<tr>
<th>Country</th>
<th>Series</th>
<th>Test</th>
<th>Test Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>$r_t$, $E_t\pi_t+12 - \bar{\pi}_t+12$, $e_t$, $y_t$</td>
<td>$Z_t$, $ADF_{GLS}$, $ADF_{GLS}$, $ADF_{GLS}$</td>
<td>-2.84, -0.69, -1.60, -3.60***</td>
</tr>
<tr>
<td>Chile</td>
<td>$r_t$, $E_t\pi_t+12 - \bar{\pi}_t+12$, $e_t$, $y_t$</td>
<td>$Z_t$, $ADF_{GLS}$, $ADF_{GLS}$, $ADF_{GLS}$</td>
<td>-1.97, -2.62, -2.41, -2.68**</td>
</tr>
<tr>
<td>Colombia</td>
<td>$r_t$, $E_t\pi_t+12 - \bar{\pi}_t+12$, $e_t$, $y_t$</td>
<td>$ADF_{GLS}$, $ADF_{GLS}$, $ADF_{GLS}$, $ADF_{GLS}$</td>
<td>-1.96, -2.50, -2.42, -6.46***</td>
</tr>
<tr>
<td>Mexico</td>
<td>$r_t$, $E_t\pi_t+12 - \bar{\pi}_t+12$, $e_t$, $y_t$</td>
<td>$ADF_{GLS}$, $ADF_{GLS}$, $ADF_{GLS}$, $ADF_{GLS}$</td>
<td>-1.23, -0.99, -2.67, -2.20**</td>
</tr>
</tbody>
</table>

Notes: $r_t$ = interest rate, $E_t\pi_t+12$ = 12-month-ahead expected inflation, $\bar{\pi}_t+12$ = 12-month-ahead inflation target, $e_t$ = nominal exchange rate, and $y_t$ = output gap. $ADF_{GLS}$ and $Z_t$ are the Elliott et al. (1996) and the Phillips and Perron (1988) tests for unit root, respectively. 

***, **, * denote statistical significance at the 1, 5 and 10% levels, respectively.
Table 2: Linear cointegration tests and estimates

A. ECM-based Cointegration tests

<table>
<thead>
<tr>
<th>Test Statistics</th>
<th>Brazil</th>
<th>Chile</th>
<th>Colombia</th>
<th>Mexico</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>-0.06</td>
<td>-0.12</td>
<td>-0.06</td>
<td>-0.14</td>
</tr>
<tr>
<td>PSS($t$-stat)</td>
<td>-4.41</td>
<td>-5.63</td>
<td>-4.13</td>
<td>-3.88</td>
</tr>
<tr>
<td>PSS(Wald)</td>
<td>5.43</td>
<td>9.25</td>
<td>5.29</td>
<td>5.42</td>
</tr>
<tr>
<td>BDM($p$-value)</td>
<td>0.02</td>
<td>&lt; 0.01</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>$-\beta_2/\lambda$</td>
<td>0.51**</td>
<td>0.63***</td>
<td>0.52***</td>
<td>0.58</td>
</tr>
</tbody>
</table>

B. Fully-Modified OLS estimates

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Brazil</th>
<th>Chile</th>
<th>Colombia</th>
<th>Mexico</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>16.69***</td>
<td>9.65**</td>
<td>10.83***</td>
<td>19.97**</td>
</tr>
<tr>
<td></td>
<td>(0.36)</td>
<td>(4.07)</td>
<td>(0.06)</td>
<td>(8.21)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-0.09***</td>
<td>-0.03***</td>
<td>-0.05***</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.01)</td>
<td>(0.001)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.59***</td>
<td>2.13***</td>
<td>0.23***</td>
<td>5.62***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.43)</td>
<td>(0.01)</td>
<td>(2.05)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>1.55***</td>
<td>-0.01</td>
<td>-0.001***</td>
<td>-1.48</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.01)</td>
<td>(0.001)</td>
<td>(0.95)</td>
</tr>
</tbody>
</table>

N. obs. 114 112 88 122

Notes: $\lambda$ is the coefficient of the lagged dependent variable (in levels) in the unrestricted error-correction equation. The number of lags for first-differenced terms in the error-correction equation was chosen on the basis of the Schwartz information criterion and the Durbin-Watson test for residual autocorrelation. PSS($t$-stat) and PSS(Wald) are the Pesaran et al. (2001) bound tests for long-run relationships based on the $t$-stat of $\lambda$ and the joint (Wald) test over long-run coefficients, respectively (Case IV as in PSS). a, b and c indicate that the statistic falls above the 5% upper bound, below the 5% lower bound and within the 5% bounds, respectively. BDM($p$-value) denotes exact $p$-values for the Banerjee et al. (1998) test of no-cointegration. The algorithm to compute exact $p$-values is provided by Ericsson and MacKinnon (2002).

Phillips and Hansen (1990) fully-modified estimations are based on the quadratic-spectral kernel and the automatic bandwidth selection method (Andrews, 1991). Standard errors are reported in parantheses. ***, ** and * denote significance at the 1, 5 and 10% levels, respectively.
Table 3: Rank test for neglected non-linear cointegration

<table>
<thead>
<tr>
<th>Country</th>
<th>Leads and lags in DOLS equation</th>
<th>Score Statistics</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>1</td>
<td>11.98</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4.22</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4.98</td>
<td>0.08</td>
</tr>
<tr>
<td>Chile</td>
<td>1</td>
<td>4.65</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3.20</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.36</td>
<td>0.51</td>
</tr>
<tr>
<td>Colombia</td>
<td>1</td>
<td>5.68</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5.87</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6.14</td>
<td>0.05</td>
</tr>
<tr>
<td>Mexico</td>
<td>1</td>
<td>6.44</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5.16</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4.27</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Notes: The Breitung (2001) test for neglected non-linearity in cointegrating vectors is used. The null hypothesis is that of linear cointegration. The score statistic is distributed as $\chi^2$ with 2 degrees of freedom.
Table 4: Non-linear cointegrating equations: Brazil, Colombia and Mexico

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Brazil (NLLS)</th>
<th>Brazil (EN-NLLS)</th>
<th>Brazil (EN-NLLS)</th>
<th>Colombia (NLLS)</th>
<th>Colombia (EN-NLLS)</th>
<th>Colombia (EN-NLLS)</th>
<th>Mexico (NLLS)</th>
<th>Mexico (EN-NLLS)</th>
<th>Mexico (EN-NLLS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>14.98***</td>
<td>15.98***</td>
<td>16.19***</td>
<td>(1.84)</td>
<td>(1.90)</td>
<td>(1.97)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>-0.14***</td>
<td>-0.14***</td>
<td>-0.14***</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{1,1}$</td>
<td>0.80***</td>
<td>0.85***</td>
<td>0.88***</td>
<td>(0.20)</td>
<td>(0.22)</td>
<td>(0.22)</td>
<td>8.95***</td>
<td>9.32***</td>
<td>9.22***</td>
</tr>
<tr>
<td>$\beta_{1,2}$</td>
<td>3.92***</td>
<td>3.56***</td>
<td>3.42***</td>
<td>(0.75)</td>
<td>(0.77)</td>
<td>(0.79)</td>
<td>0.003***</td>
<td>0.003***</td>
<td>0.003***</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>26.33***</td>
<td>27.68***</td>
<td>28.09***</td>
<td>(7.49)</td>
<td>(7.71)</td>
<td>(8.09)</td>
<td>8.49***</td>
<td>8.21***</td>
<td>8.13***</td>
</tr>
<tr>
<td>$\beta_{2,1}$</td>
<td>-8.95***</td>
<td>-9.32***</td>
<td>-9.22***</td>
<td>(2.74)</td>
<td>(2.69)</td>
<td>(2.62)</td>
<td>-18.71***</td>
<td>-17.81***</td>
<td>-17.57***</td>
</tr>
<tr>
<td>$\beta_{2,2}$</td>
<td>-2.33***</td>
<td>-2.31***</td>
<td>-2.24***</td>
<td>(0.36)</td>
<td>(0.36)</td>
<td>(0.36)</td>
<td>-0.009***</td>
<td>-0.01***</td>
<td>-0.01***</td>
</tr>
<tr>
<td>$c$</td>
<td>-0.27***</td>
<td>-0.30***</td>
<td>-0.29***</td>
<td>(0.10)</td>
<td>(0.11)</td>
<td>(0.05)</td>
<td>0.26***</td>
<td>0.24***</td>
<td>0.23***</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>4.23***</td>
<td>3.85***</td>
<td>3.91***</td>
<td>(2.15)</td>
<td>(1.83)</td>
<td>(1.97)</td>
<td>3.87***</td>
<td>3.23***</td>
<td>2.90***</td>
</tr>
<tr>
<td>N. obs.</td>
<td>114</td>
<td>112</td>
<td>111</td>
<td>88</td>
<td>86</td>
<td>85</td>
<td>122</td>
<td>120</td>
<td>119</td>
</tr>
<tr>
<td>$\ell$</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Notes: Approximated standard errors are reported in parentheses. $\ell$ is the number of lags of innovations in the EN-NLLS correction procedure. Wald test statistics for linear restrictions are reported along with $p$-values (in brackets).

***, ** and * denote statistical significance at the 1, 5 and 10% levels, respectively.
Table 5: Choi and Saikkonen (2010) test for non-linear cointegration

A. NLLS Residuals

<table>
<thead>
<tr>
<th>Country</th>
<th>Rule</th>
<th>Bandwidth</th>
<th>Block Size</th>
<th>M</th>
<th>$C^{b,\text{max}}_{\text{NLLS}}$</th>
<th>p-value</th>
<th>5% level $(\alpha_M)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>FX</td>
<td>Automatic</td>
<td>71</td>
<td>2</td>
<td>2.41</td>
<td>0.02</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>Fixed</td>
<td></td>
<td>71</td>
<td>2</td>
<td>1.69</td>
<td>0.05</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>MV</td>
<td>Automatic</td>
<td>60</td>
<td>2</td>
<td>0.22</td>
<td>0.59</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>Fixed</td>
<td></td>
<td>54</td>
<td>3</td>
<td>0.15</td>
<td>0.32</td>
<td>0.017</td>
</tr>
<tr>
<td>Colombia</td>
<td>FX</td>
<td>Automatic</td>
<td>57</td>
<td>2</td>
<td>0.86</td>
<td>0.17</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>Fixed</td>
<td></td>
<td>57</td>
<td>2</td>
<td>0.66</td>
<td>0.24</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>MV</td>
<td>Automatic</td>
<td>47</td>
<td>2</td>
<td>1.56</td>
<td>0.06</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>Fixed</td>
<td></td>
<td>52</td>
<td>2</td>
<td>1.05</td>
<td>0.12</td>
<td>0.025</td>
</tr>
<tr>
<td>Mexico</td>
<td>FX</td>
<td>Automatic</td>
<td>76</td>
<td>2</td>
<td>2.27</td>
<td>0.02</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>Fixed</td>
<td></td>
<td>76</td>
<td>2</td>
<td>1.58</td>
<td>0.26</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>MV</td>
<td>Automatic</td>
<td>61</td>
<td>2</td>
<td>1.45</td>
<td>0.07</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>Fixed</td>
<td></td>
<td>72</td>
<td>2</td>
<td>1.82</td>
<td>0.04</td>
<td>0.025</td>
</tr>
</tbody>
</table>

B. EN-NLLS Residuals ($\ell = 1$)

<table>
<thead>
<tr>
<th>Country</th>
<th>Rule</th>
<th>Bandwidth</th>
<th>Block Size</th>
<th>M</th>
<th>$C^{b,\text{max}}_{\text{EN-NLLS}}$</th>
<th>p-value</th>
<th>5% level $(\alpha_M)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>FX</td>
<td>Automatic</td>
<td>70</td>
<td>2</td>
<td>2.40</td>
<td>0.02</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>Fixed</td>
<td></td>
<td>70</td>
<td>2</td>
<td>1.70</td>
<td>0.05</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>MV</td>
<td>Automatic</td>
<td>59</td>
<td>2</td>
<td>0.21</td>
<td>0.61</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>Fixed</td>
<td></td>
<td>42</td>
<td>3</td>
<td>0.35</td>
<td>0.44</td>
<td>0.017</td>
</tr>
<tr>
<td>Colombia</td>
<td>FX</td>
<td>Automatic</td>
<td>56</td>
<td>2</td>
<td>0.87</td>
<td>0.17</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>Fixed</td>
<td></td>
<td>56</td>
<td>2</td>
<td>0.63</td>
<td>0.25</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>MV</td>
<td>Automatic</td>
<td>46</td>
<td>2</td>
<td>1.57</td>
<td>0.06</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>Fixed</td>
<td></td>
<td>24</td>
<td>4</td>
<td>0.82</td>
<td>0.18</td>
<td>0.012</td>
</tr>
<tr>
<td>Mexico</td>
<td>FX</td>
<td>Automatic</td>
<td>75</td>
<td>2</td>
<td>2.48</td>
<td>0.02</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>Fixed</td>
<td></td>
<td>75</td>
<td>2</td>
<td>1.73</td>
<td>0.04</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>MV</td>
<td>Automatic</td>
<td>42</td>
<td>3</td>
<td>2.17</td>
<td>0.02</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>Fixed</td>
<td></td>
<td>42</td>
<td>3</td>
<td>1.50</td>
<td>0.06</td>
<td>0.017</td>
</tr>
</tbody>
</table>

Notes: The null hypothesis is that of non-linear cointegration (stationary residuals). FX and MV denote the fixed and minimum volatility rules, respectively. The long-run variance is estimated through the Quadratic Spectral kernel (Andrews, 1991; Andrews and Monahan, 1992). “Automatic” denotes the data-dependent lag-length selection method suggested by Andrews (1991). “Fixed” denotes the lag-length selection method suggested by Kwiatkowski et al. (1992), with fixed bandwidth $(4b/100)^{0.25}$. The criterion for non-rejection of the null at the 5% level of significance is $p$-value > Adjusted 5% level $(\alpha_M)$. 